An International Journal
computers & mathematics with applications

PERGAMON Computers and Mathematics with Applications 39 (2000) 121–125
www.elsevier.nl/locate/camwa

Maximum Likelihood Global Positioning System Receiver

W. J. HURD
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109, U.S.A.

Abstract—A maximum likelihood receiver concept has been developed for measuring the positions of vehicles using the signals from the Global Positioning System satellites. This receiver enables tracking when the vehicles are undergoing severe dynamics, including accelerations of 150 g and jerk of 150 g/s, as may be encountered in range testing of aircraft and missiles.

1. INTRODUCTION

This paper presents an example of doing productive research using a fundamental principle taught to me by our mentor, Professor Golomb. This principle is that productive research often is not difficult for the researcher; instead, it is accomplished by doing something which is easy for the researcher, but which has not been done before and solves an important problem. The problem in this example is measuring the position of a Global Positioning System (GPS) receiver which is undergoing very high dynamics.

2. PROBLEM DESCRIPTION

The GPS is a network of satellites which transmit range codes modulated by data, both phase modulated on L-band RF carriers. A receiver located almost anywhere on Earth can determine its position to within a few meters by measuring the range code delay (or phase) from four or more satellites.

Traditional radio-navigation receivers phase lock to the received carrier signals. In the GPS case, this enables extraction of the data modulation as well as the navigation parameters. Phase-locked receivers work well when the receiver dynamics are low, because a narrow bandwidth phase-locked loop can be used to track the carrier, with a high loop signal-to-noise ratio. In the GPS case, phase-locked receivers work for dynamics up to approximately 5 g in acceleration and 5 g/s in jerk. Above these approximate limits, a loop wide enough to track the dynamics will be too wide to achieve an adequate signal-to-noise ratio.

The key steps in developing the high dynamics receiver concept were to recognize that it is not necessary to extract the data while under high dynamics, and that range and frequency

The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, and was sponsored by the United States Air Force Systems Command through an agreement with the National Aeronautics and Space Administration, under task order RE-182, Amendment 452.

0898-1221/00/$ - see front matter Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX
PII: S0898-1221(00)00117-6

(range rate) are adequate for positioning. Therefore, it is not necessary to track carrier phase. A receiver performing maximum-likelihood estimation (MLE) of carrier frequency and range code delay would suffice.

3. SIGNAL DESCRIPTION AND MLE SOLUTION

This section discusses the approximate characteristics of the signals from the GPS satellites, and the maximum likelihood estimation of pseudorange and range rate from these signals. Details are given in references [1–3].

A. GPS Signal Characteristics

The GPS satellites transmit pseudonoise signals at two L-band frequencies denoted L1 and L2. Two different pseudonoise signals are used on each satellite, a P code signal and a C/A code signal. The P code is a long period pseudonoise sequence with a chip (clock) rate of 10.23 MHz, and the C/A code is a period 1023 pseudonoise code clocked at 1.023 MHz. Both code signals are biphase modulated by the same binary data at 50 bps. The L1 carrier is phase modulated by both pseudonoise signals, with the C/A code lagging the P code by 90 degrees. The L2 carrier is modulated by either the P or the C/A signal, but not both at the same time. The L1 carrier frequency is 154 times the P code chip rate, the L2 carrier frequency is 120 times the P code chip rate, and all frequencies are phase coherent. On L1, the C/A signal has twice the power of the P code signal. The carriers are completely suppressed.

The L1 signal can be expressed as

$$A_1 D(t) p(t) \cos(\omega_1 t + \phi_1) + \sqrt{2}\, A_1 D(t) c(t) \sin(\omega_1 t + \phi_1)\,, \tag{1}$$

where

A_1 = P signal amplitude,
$D(t)$ = data signal at 50 bps,
$p(t)$ = P code signal,
$c(t)$ = C/A code signal,
ω_1 = L1 radian carrier frequency,
ϕ_1 = constant but random carrier phase.

B. Maximum Likelihood Estimation

A receiver receives the transmitted signal delayed by the path length between satellite and receiver, Doppler shifted due to relative velocity, attenuated, and with noise added. The receiver has a different clock than the satellite, and by observing the signal from only one satellite, it cannot tell the difference between the clock offset and the signal delay due to range.

The received L1 signal $r(t)$ is

$$r(t) = AD(t-\tau)\left[p(t-\tau)\cos(\omega(t-\tau)+\phi) + \sqrt{2}\, c(t-\tau)\sin(\omega(t-\tau)+\phi)\right] + n(t), \tag{2}$$

where

A = amplitude of received L1, P signal,
τ = time delay plus clock offset, proportional to pseudorange,
ϕ = phase of received L1 signal suppressed carrier,
$n(t)$ = white Gaussian noise,
$D(\bullet)$ = binary data modulation.

Now assume that the signal is observed over a period of time T, such as one data bit time, during which all parameters A, D, ω, and ϕ, are constant. The parameters of interest for tracking are ω and τ, and their maximum likelihood estimates are the values of τ_m and ω_m which maximize

$$\left| \int_T r(t)\left[p(t-\tau_m) + j\sqrt{2}\, c(t-\tau_m)\right] e^{-j\omega_m t}\, dt \right|^2 . \tag{3}$$

4. IMPLEMENTATION

This MLE solution is implemented approximately by performing cross correlations for a finite set of τ_m, followed by fast Fourier transforms, choosing the τ_m and ω_m which maximize expression (3), and then interpolating to improve the resolution. This process is repeated every data bit time, or 50 times per second. The delay and frequency parameters are then tracked with control loops.

5. PERFORMANCE DEMONSTRATION

A breadboard receiver was built to demonstrate the high dynamic tracking ability, using a simulated P code only. Analyses, simulations, and experimental results are documented elsewhere [3,4].

The performance of the breadboard receiver was demonstrated by tracking signals simulating what would be received by a receiver moving over a circular path. The resulting velocity, acceleration, and jerk are sinusoidal, and can be characterized by peak acceleration and peak jerk. Figures 1–3 summarize the theoretical, experimental, and simulation results. As predicted, thresholding occurs at SNRs below approximately 32 dB-Hz, or 15 dB in the data bit time of 20 ms. This is due to the occurrence of outliers, or false alarms in the MLE. At high SNR, the observed errors are higher than predicted from noise analysis, due to instrumentation effects such as interpolation between FFT frequencies and correlator lags. Tracking was successful and in agreement with theory for accelerations as high as 150 g and jerk as high as 157 g/s.

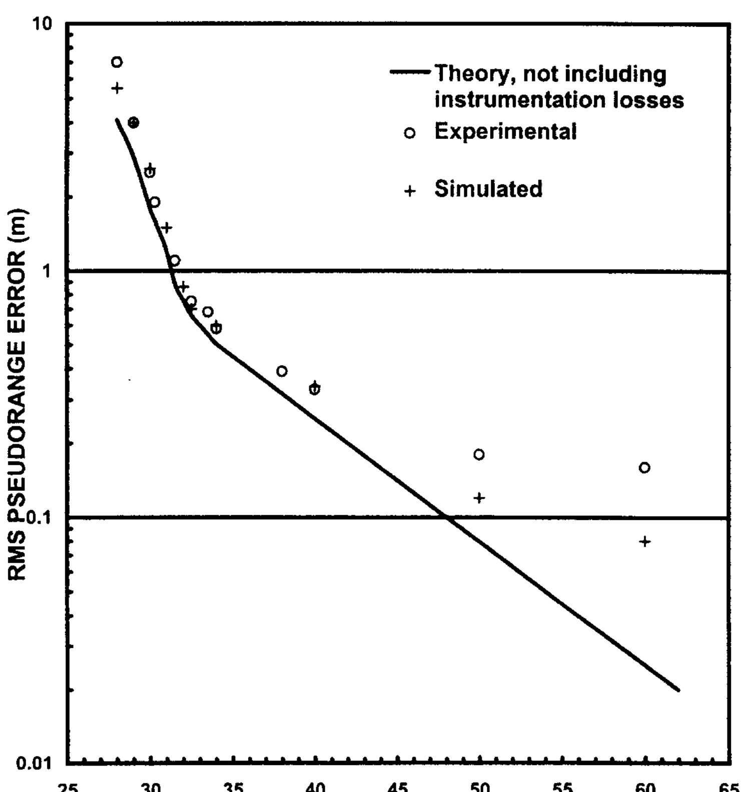

Figure 1. Pseudorange results for 50 g amd 40 g/s.

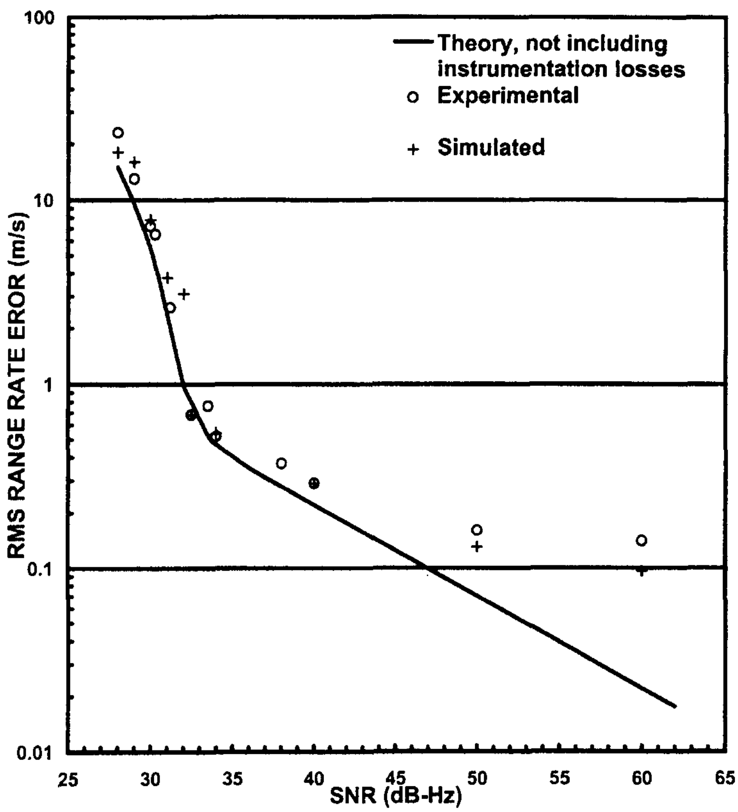

Figure 2. Range rate results for 50 g amd 40 g/s.

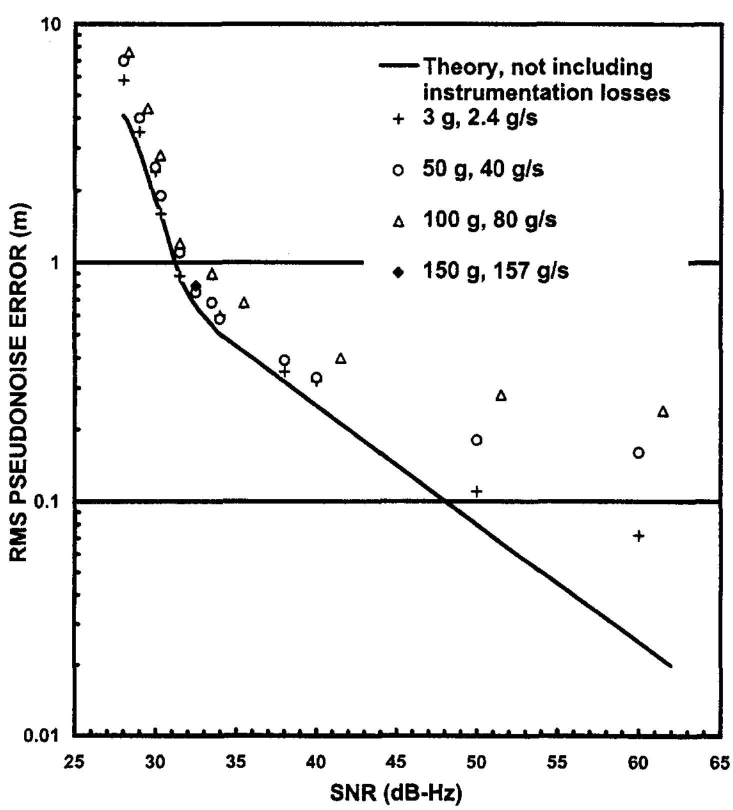

Figure 3. Pseudorange results versus dynamics.

6. CONCLUSION

This paper presents an example of productive research accomplished without difficult or revolutionary theory. Instead, an important problem was solved by use of well-known techniques which had not previously been applied to the problem. This "easy" approach to research is an approach learned from Professor Golomb.

REFERENCES

1. W.J. Hurd, High dynamic, low volume GPS receiver, *IEEE 1983 National Telesystems Conference*, San Francisco, November 1983, pp. 219–225.
2. W.J. Hurd and J.I. Statman, High dynamic GPS unaided pseudorange tracking demonstration, *International Telemetering Conference*, Las Vegas, NV, October 22–25, 1984, p. 389.
3. W.J. Hurd, J.I. Statman and V.A. Vilnrotter, High dynamic GPS receiver using maximum likelihood estimation and frequency tracking, *IEEE Transactions on Aerospace and Electronic Systems* **AES-23** (4), 425–437, (July 1987).
4. W.J. Hurd, J.I. Statman and V.A. Vilnrotter, High dynamic GPS receiver validation demonstration, Final Report, Publication 85-74, Jet Propulsion Laboratory, Pasadena, CA, (October 31, 1985).